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Full Waveform Inversion Method for Horizontally Inhomogeneous Stratified Media

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Abstract: Full waveform inversion method is an approach to grasp the physical property parameters of underground media in geotechnical nondestructive detection and testing field. Using finite-difference time domain (FDTD) method for elastic wave equations, the full-wave field in horizontally inhomogeneous stratified media for elastic wave logging was calculated. A numerical 2D model with three layers was computed for elastic wave propagation in horizontally inhomogeneous media. The full waveform inversion method was verified to be feasible for evaluating elastic parameters in lateral inhomogeneous stratified media and showed well accuracy and convergence. It was shown that the time cost of inversion had certain dependence on the choice of starting initial model. Furthermore, this method was used in the detection of nonuniform grouting in the construction of immersed tube tunnel. The distribution of nonuniform grouting was clearly evaluated by the S-wave velocity profile of grouted mortar base below the tunnel floor.

Key words: full waveform inversion, stratified media, horizontally inhomogeneity, elastic parameter
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0 Introduction

Full waveform inversion method was first applied in geophysics and seismology to identify the physical property parameters of materials inside the crust, especially the densities and velocities[1]. It has been gradually adopted in geotechnical detection and testing field since the 1990s, and is drawn more and more attention now. It attempts to match the actual parameters by some iterative approaches utilizing the amplitude, travel time and phase of measured waveforms comprehensively[2].

Elastic wave detection is a feasible and nondestructive method in geotechnical detection and testing field. When the elastic wave passes through target medium, the physical property information of materials will be carried by waveforms and received by detectors[3-4]. By received waveform data, the full waveform inversion method is used to work out the various elastic parameters of target medium, such as P-wave velocity, S-wave velocity and medium density; these parameters are helpful for lithology recognition[5-7]. Meanwhile, many geotechnical media can be simplified by stratified media, and based on this, Grechka et al.[8-9] successfully reached the anisotropy parameters of soil medium combining the P-wave and pulse seismic wave inversion.

The framework of full waveform inversion method mainly contains three critical issues: full waveform data acquisition, forward computation of full-wave field and inversion approach[10]. In this paper, the full waveform data acquired in site were improved by some necessary waveform pretreatments, such as signal-to-noise ratio enhancement, filtering and normalization. The finite-difference time domain (FDTD) method was used for solution of elastic wave equations in forward computation[11] and the least square method was adopted in inverse computation[12]. Meanwhile, a numerical 2D model with three layers was computed for elastic wave propagation in horizontally inhomogeneous media. The feasibility of this method for revealing elastic parameters in numerical stratified model was verified. Based on the numerical results, the real-data case studies where nonuniform grouting occurred in the construction of immersed tube tunnel were discussed. The S-wave velocity profile of grouted mortar base below the tunnel floor was obtained by the full waveform inversion method using the recorded elastic wave loggings. As a result, the effect of grouting was clearly evaluated.

1 Full Waveform Inversion Method

1.1 Theoretical Analysis

For a stratified half-space, inputted by a given

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excitation, the response elastic wave field can be evaluated by solution of the FDTD method. Response waveforms at specific points are recorded and denoted as a matrix $\boldsymbol{F}$. The statement of forward calculation is

$$\boldsymbol{F} = f(\boldsymbol{x}), \tag{1}$$

where f is a function of forward calculation, and $\boldsymbol{x}$ is the parameter vector to be grasped. Generally, the function of forward calculation is nonlinear and can be simplified to linearization by Born approximate criterion in the full waveform inversion[13].

According to the observation waveform matrix $\boldsymbol{F}_{\text{obs}}$ in site and relevant auxiliary information, an initial stratified model with parameter vector $\boldsymbol{x}_{\text{ini}}$ can be sought. Then the initial response waveform matrix $\boldsymbol{F}_{\text{ini}}$ is obtained by forward calculation. In general, $\boldsymbol{F}_{\text{obs}} \neq \boldsymbol{F}_{\text{ini}}$, so we define a residual matrix $\boldsymbol{\varepsilon}$ as

$$\boldsymbol{\varepsilon} = \boldsymbol{F}_{\text{obs}} - \boldsymbol{F}_{\text{ini}}. \tag{2}$$

Assumed that there exits an appropriate parameter vector $\boldsymbol{x}$ which makes the response waveform matrix $\boldsymbol{F}$ as close as possible to the observation waveform matrix $\boldsymbol{F}_{\text{obs}}$. Thus, $\boldsymbol{F}$ can be approximated by the first Taylor series expansion:

$$\boldsymbol{F} = \boldsymbol{F}_{\text{ini}} + \sum_{i=1}^{n} \frac{\partial \boldsymbol{F}}{\partial x_i} \Delta x_i, \tag{3}$$

where n is the number of objective parameters, x_i is the ith objective parameter in parameter vector $\boldsymbol{x}$, and Δx_i is the corresponding adjust value. The partial derivative parameter $\partial \boldsymbol{F}/\partial x_i$ is sensitivity to response waveform matrix $\boldsymbol{F}$.

Meanwhile, assumed that the waveform matrices $\boldsymbol{F}_{\text{obs}}$, $\boldsymbol{F}_{\text{ini}}$ and $\boldsymbol{F}$ are composed of j vectors. We can define 2-norm value of the difference between $\boldsymbol{F}_{\text{obs}}$ and $\boldsymbol{F}$ as an objective function Q, which is expressed as

$$Q = \|\boldsymbol{F}_{\text{obs}} - \boldsymbol{F}\|_2 = \sum_{j=1}^{m} \|\boldsymbol{F}_{\text{obs},j} - \boldsymbol{F}_j\|_2, \tag{4}$$

where $\|\cdot\|_2$ is the 2-norm operator, $\boldsymbol{F}_j$ and $\boldsymbol{F}_{\text{obs},j}$ are the jth vectors of corresponding matrix, and m is the number of observation points. Inserted Eqs. (2) and (3) into Eq. (4), the detailed expression of Q is shown in

$$Q = \sum_{j=1}^{m} \left\| \sum_{i=1}^{n} \frac{\partial \boldsymbol{F}_j(t)}{\partial x_i} \Delta x_i - \boldsymbol{\varepsilon}_j \right\|_2, \tag{5}$$

where $\boldsymbol{\varepsilon}_j$ is the jth vector of residual matrix $\boldsymbol{\varepsilon}$.

As objective function Q gets its minimum, the most appropriate parameter vector $\boldsymbol{x}$ is reached. In this state by least square method, the partial derivatives of Q are equal to zero, i.e.,

$$\frac{\partial Q}{\partial \Delta x_i} = \sum_{j=1}^{m} \left[\left(\sum_{i=1}^{n} \frac{\partial \boldsymbol{F}_j(t)}{\partial x_i} \Delta x_i - \boldsymbol{\varepsilon}_j \right) \cdot \frac{\partial \boldsymbol{F}_j(t)}{\partial x_i} \right] = 0, \tag{6}$$

where "·" is the dot product operator. The Eq. (6) is a system of linear equations and can be rewritten in matrix form as

$$\boldsymbol{A} \Delta \boldsymbol{x} = \boldsymbol{B}, \tag{7}$$

where $\boldsymbol{A}$ is the Jacobian matrix with size of $n \times n$, $\boldsymbol{B}$ is a vector with size of $n \times 1$, and $\Delta \boldsymbol{x}$ is adjust vector to be solved. Element $A_{p,l}$ in Jacobian matrix $\boldsymbol{A}$ is calculated by

$$A_{p,l} = \sum_{j=1}^{m} \left(\frac{\partial \boldsymbol{F}_j(t)}{\partial x_p} \cdot \frac{\partial \boldsymbol{F}_j(t)}{\partial x_l} \right), \quad p, l = 1, 2, \cdots, n,$$

and element B_p in vector $\boldsymbol{B}$ is calculated by

$$B_p = \sum_{j=1}^{m} \left(\frac{\partial \boldsymbol{F}_j(t)}{\partial x_p} \cdot \boldsymbol{\varepsilon}_j \right), \quad p = 1, 2, \cdots, n.$$

For it is difficult or unable to get explicit expression of forward function f, the derivative $\partial \boldsymbol{F}_j(t)/\partial x_i$ inside the matrix $\boldsymbol{A}$ and vector $\boldsymbol{B}$ is replaced by differential quotient, i.e.,

$$\frac{\partial \boldsymbol{F}_j(t)}{\partial x_i} = \frac{f(\boldsymbol{x} + \delta \boldsymbol{x}) - f(\boldsymbol{x})}{\delta x_i}, \tag{8}$$

where

$$\delta \boldsymbol{x} = [0 \quad \cdots \quad \delta x_i \quad \cdots \quad 0]^{\mathrm{T}}.$$

It is a slight disturbance on the ith objective parameter x_i in parameter vector $\boldsymbol{x}$. Generally, δx_i is taken as 5%—10% of x_i.

As Jacobian matrix $\boldsymbol{A}$ is symmetrical positive, so the solution of linear equation Eq. (7) is

$$\Delta \boldsymbol{x} = \boldsymbol{A}^{-1} \boldsymbol{B}. \tag{9}$$

After that, by adding the adjust vector $\Delta \boldsymbol{x}$, the new parameter vector $\boldsymbol{x}$ is modified as $\boldsymbol{x} + \Delta \boldsymbol{x}$.

By repeating the above calculation until the objective function Q reaches the terminal condition. Eventually, the latest parameter vector $\boldsymbol{x}$ is output as the final result. In this work, the terminal condition is that the value of objective function Q is less than 10^{-3}.

1.2 Steps of Inversion Computation

According to the theoretical arithmetic mentioned in Subsection 1.1, the computation of full waveform inversion method can be divided into the following steps.

(1) Seek a starting initial model $\boldsymbol{x}_{\text{ini}}$ on basis of observation waveform matrix $\boldsymbol{F}_{\text{obs}}$ and auxiliary information in site.

(2) On the basis of initial model $\boldsymbol{x}_{\text{ini}}$, carry out the forward calculation by FDTD method, and obtain the response waveform matrix $\boldsymbol{F}_{\text{ini}}$.

(3) Calculate the residual matrix $\boldsymbol{\varepsilon}$, as well as the value of objective function Q;

(4) Ascertain whether the terminal condition is satisfied. If not, by solution of Eqs. (7) and (8), get adjust vector $\Delta\boldsymbol{x}$.

(5) Modify the parameter vector $\boldsymbol{x}$ by $\boldsymbol{x} + \Delta\boldsymbol{x}$, and repeat Steps (2)—(4).

(6) If the terminal condition is satisfied, break out the calculation and output the latest parameter vector $\boldsymbol{x}$ as the final result.

2 Numerical Example

In this section, the full waveform inversion method for horizontally inhomogeneous stratified media would be testified by a numerical 2D model.

2.1 Modeling

By a similar grouting construction of immersed tube tunnel, we cast the numerical 2D stratified model with three layers. The materials of top and bottom layers were concrete and bearing stratum, whose physical property parameters were known. The intermediate layer was defect layer and the S-wave velocities were unknown. In view of the horizontally inhomogeneity, the defect layer was simply distinguished by four different elastic regions and represented by different S-wave velocities, which are v_{s1}, v_{s2}, v_{s3} and v_{s4} (from left to right), as shown in Fig. 1. In the model, the upper boundary condition was set as free while the left boundary condition was set as perfect matched layer (PML) absorbing boundary[14], as well as the right and the bottom side.

The Ricker wavelet with center frequency of 1 kHz, as shown in Fig. 2, was used as excitation and inputted on the surface of model whose shot interval was 0.25 m. The time-history of vertical displacements on four reception points at surface was chosen as response waveform. The interval of reception points was 0.25 m. In the process of full waveform inversion, all of the materials in the numerical stratified model were assumed as isotropic elastic media and the stress and strain were kept in elastic range. Their physical property parameters were listed in Table 1, where ρ and v_{s} are the density and S-wave velocity of each material respectively, h is the thickness of each layer and μ is the Poisson's ratio.

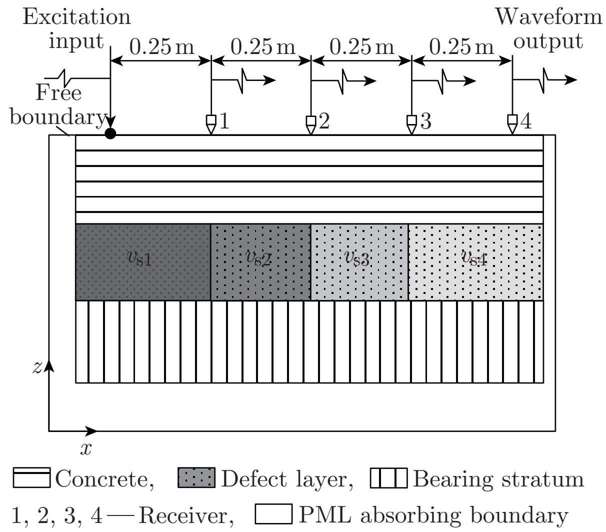

Fig. 1 Numerical stratified model

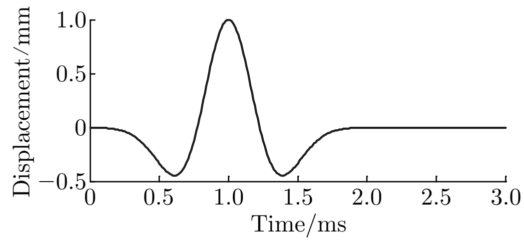

Fig. 2 Waveform of inputted Riker wavelet

Table 1 Material parameters in numerical model

Layer	$\rho/(\text{g}\cdot\text{m}^{-3})$	$v_{\text{s}}/(\text{km}\cdot\text{s}^{-1})$	h/m	μ
Concrete	2.4	2.5	1.4	0.2
Defect layer	1.8	unknown	0.6	0.3
Bearing stratum	2.0	1.5	2.6	0.3

2.2 Inversion on Numerical Model

At first, the S-wave velocities in defect layer were preset as (from left to right) $v_{\text{s1}} = 1.500\,\text{km/s}$, $v_{\text{s2}} = 0.800\,\text{km/s}$, $v_3 = 0.200\,\text{km/s}$ and $v_{\text{s4}} = 1.500\,\text{km/s}$, respectively. Then by the forward calculation, the response waveform vectors at four reception points were considered as observation waveform matrix $\boldsymbol{F}_{\text{obs}}$.

Then the S-wave velocities above were reset as unknown parameters to be grasped. Choose starting initial model $\boldsymbol{x}_{\text{ini}}$ as $v_{\text{s1}} = 0.600\,\text{km/s}$, $v_{\text{s2}} = 0.600\,\text{km/s}$, $v_{\text{s3}} = 0.600\,\text{km/s}$ and $v_{\text{s4}} = 0.600\,\text{km/s}$; the full waveform inverse calculation was carried out. The final results were $v_{\text{s1}} = 1.497\,\text{km/s}$, $v_{\text{s2}} = 0.800\,\text{km/s}$, $v_{\text{s3}} = 0.201\,\text{km/s}$ and $v_{\text{s4}} = 1.502\,\text{km/s}$. Compared with the preset values, error of each S-wave velocity was nearly zero, and the final waveforms almost coincided with the observation, as shown in Fig. 3.

From the above details, the full waveform inversion

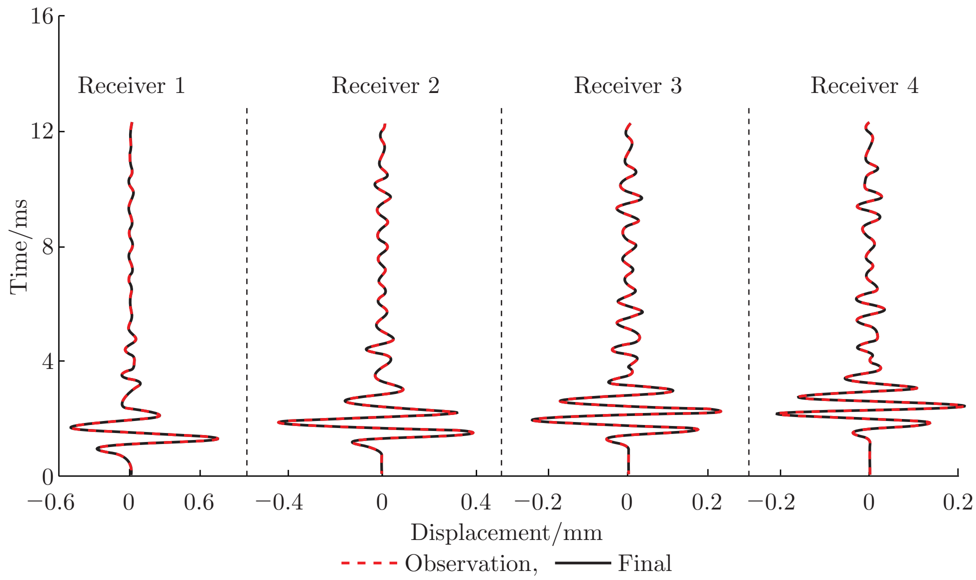

Fig. 3 Comparison between observation waveforms and final waveforms

method was verified to be feasible for revealing S-wave velocities in horizontally inhomogeneous stratified media. The accuracy and convergence of this method are very well even if the starting initial model is too far away from the actual values. But it must be pointed out that the time cost of inversion has certain of dependence on the choice of starting initial model, so in actual projects, it is advisable that the starting initial model should be determined with the help of auxiliary information as far as possible.

3 Application on Field Detection

In this section, the full waveform inversion method for horizontally inhomogeneous stratified media was applied in the grouting detection in an immersed tube tunnel project under construction. The test site was located in the tail end of tunnel. When field detection was carried out on the surface of tunnel floor, the grouting construction had not been all accomplished, thus mortar base at there was not completely dense and water was gushing outside from the pass-ways in the mortar base under tunnel floor. Pretreated by some approaches, the collected waveform data on measure lines were adopted as observation waveform matrix $\boldsymbol{F}_{\rm obs}$. And then by the full waveform inversion method, the S-wave profile under the tunnel floor was reached eventually, which was contributed to finding out weak space and pass-ways in mortar base. Moreover, it would provide important reference for subsequent grouting construction on blocking water gush and reinforcing mortar base.

3.1 General

The proposed immersed tube tunnel was located in seismic zone, so in order to satisfy seismic resistance, the sandy foundation generally used in immersed tube tunnel was replaced by grouted mortar base. Before grouting construction, as the tunnel moved into the predetermined location and designed elevation, an intermediate space would be preserved between the undersurface of tunnel floor and the flattened gravel trench and it was full of saturated soil usually. By grouting construction, the intermediate space was full filled by mortar. When the mortar layer solidified, the immersed tube tunnel would lay on the mortar base.

Similar to the numerical example in Subsection 2.1, a stratified horizontally inhomogeneous 2D model with three layers was established like Fig. 1. The first layer was tunnel floor consisted of concrete; the intermediate layer was mortar base layer; the bottom layer was flattened gravel trench. In this stratified model, all the physical property parameters were similar to the corresponding numerical materials listed in Table 1, as well as the boundary conditions and the computation assumption.

3.2 Waveform Acquisition

During the waveform acquisition work, all of the used devices were personal computer, small seismograph, receivers, power supply of 12 V, hammer, cables and so on, as shown in Fig. 4. During the acquisition work,

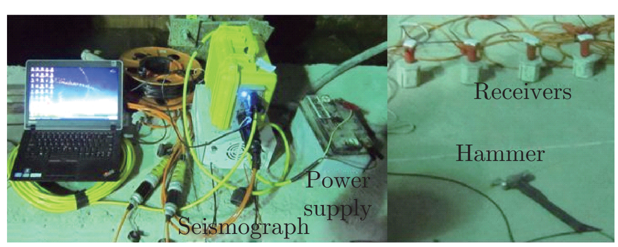

Fig. 4 Equipments for acquisition work in field

a round head hammer with mass of about 0.68 kg was used to strike the surface of tunnel floor like a hypocenter. Vertical vibration signals were detected by four receivers (the shot interval and the receivers interval were both 0.25 m). Time-history curves of vertical velocities were recorded by small seismograph. As the data acquisition work was completed on a point, the above process would be repeated on next point until all the points on entire measure line were accomplished, as shown in Fig. 5.

Because the various background noises were generated by construction equipment and ship traffic and most of them were low spectral characteristic, the acquired waveform should be filtered by frequency band-pass in band of 0.3—5.0 kHz. Moreover, as the hammer was manually stroked, the energy and distance of each excitation were uncontrollable and imprecise, in order to eliminate these influences, normalization on the amplitude and phase was applied to acquired waveform. Finally by the above pretreatments, the acquired waveform was improved, as shown in Fig. 6. They would be considered as $\boldsymbol{F}_{\text{obs}}$ for the following inverse calculation.

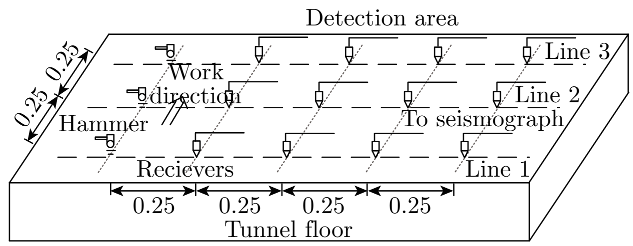

Fig. 5 Sketch of acquisition work in field (m)

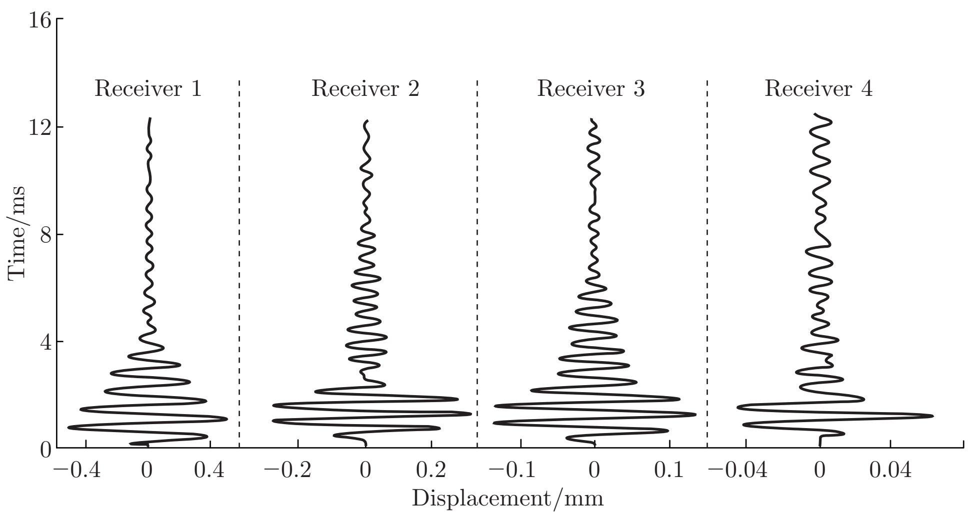

Fig. 6 Time-history curves of improved acquired waveform

3.3 Inversion Results

According to the results of geological survey, the S-wave velocity in saturated soil was around 0.300 km/s and it was about 1.200 km/s in motor base by conservative estimates, so the initial S-wave velocities of mortar base to be inversed were preset as $v_{si} = 0.700$ km/s ($i = 1, 2, 3, 4$). For measure Line 1, during inverse computation, the value of objective function Q was reduced, as shown in Fig. 7. When the value reached terminal condition stably, reverse computation broke out and the results of S-wave velocities were $v_{s1} = 1.302$ km/s, $v_{s2} = 1.150$ km/s, $v_{s3} = 1.207$ km/s and $v_{s4} = 1.332$ km/s.

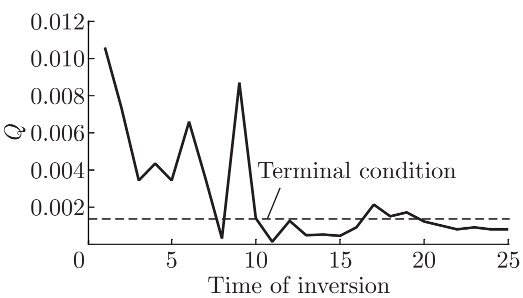

Fig. 7 Value of function Q reduced by inversion steps

Repeat above process on other measure lines until all of them in detection area were finished. At last the S-wave velocity profile was reached, as shown in Fig. 8. The x-axis presented north-south (NS) direction of the tunnel and the y-axis presented east-west (EW) direction. As a result, distribution of nonuniform grouting was clearly evaluated by the S-wave velocity profile, which showed good agreement with the gushing in site. Also, the pass-ways and weak space possibly exited in mortar base were detected. It would provide important reference for subsequent grouting construction on blocking water gush and reinforcing mortar base.

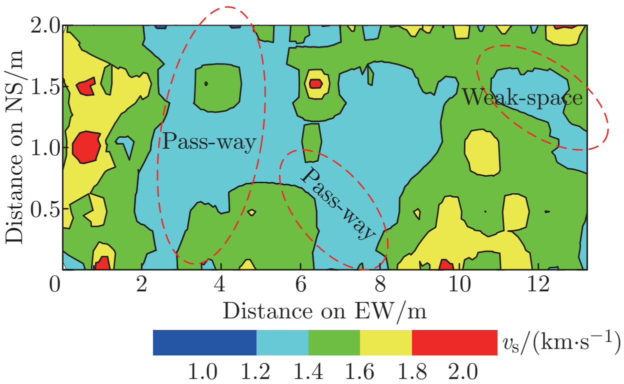

Fig. 8 S-wave velocity profile inversed by waveform inversion method

4 Conclusion

It is verified that the full waveform inversion method is available to grasp elastic parameters in horizontally inhomogeneous stratified media in geotechnical field, using the FDTD method in forward calculation and the least squares method in the inversion analysis.

The accuracy and convergence of full waveform inversion method are reliable. It shows that the time cost is depended on the choice of starting initial model.

The S-wave velocity profile of grouted mortar base below the tunnel floor was obtained by full waveform inversion method. As a result, the distribution of nonuniform grouting was clearly evaluated, and it shows good agreement with the water pass-way in the site.

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